International Journal of Theoretical Physics, Vol. 41, No. 2, February 2002 (© 2002)

Light-Front Hamiltonian and BRST Formulations of a Two-Dimensional Abelian Higgs Model in the Broken Symmetry Phase

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Received April 2, 2001

The light-front Hamiltonian and BRST formulations of an abelian Higgs model involving the electromagnetic vector gauge field are investigated in one space, one time dimension in the broken symmetry phase, where the phase $\phi(x, t)$ of the complex matter field $\Phi(x, t)$ carries the charge degree of freedom of the complex matter field and is, in fact, akin to the Goldstone boson.

1. INTRODUCTION

The models of quantum electrodynamics with a Higgs potential, namely, the abelian Higgs models involving the vector gauge field $A^\mu(x^\mu)$ in lower (one space, one time (1+1)- or two space, one time (2+1)-) dimensions have been of wide interest in the recent years (Abrikosov, 1957a,b; Banerjee *et al.*, 1995, 1997; Banks and Lykken, 1990; Bogomol'nyi, 1976a,b; Chen *et al.*, 1989; Daser *et al.*, 1982a,b; De Vega and Schaposnik, 1976; Dunne and Trugenberger, 1991; Fetter *et al.*, 1989; Forte, 1992; Friedberg and Lee, 1977a,b, 1978; Ginsburg and Landau, 1950; Jackiw, 1989; Jackobs and Rebbi, 1986; Krive and Rozhavskii, 1987; Kulshreshtha, in press-a; Laughlin, 1988; Lee *et al.*, 1991; Lee and Nam, 1991; Mac Kenzie and Wilczek, 1988; Nielsen and Olesen, 1973a,b; Saint-James *et al.*, 1969). These models involving a Maxwell term, which accounts for the kinetic energy of the vector gauge field $A^\mu(x^\mu)$ (Abrikosov, 1957a,b; Banks and Lykken, 1990; Bogomol'nyi, 1976a,b; Chen *et al.*, 1989; De Vega and Schaposnik, 1976; Fetter *et al.*, 1989; Forte, 1992; Friedberg and Lee, 1977a,b, 1978; Ginsburg and Landau, 1950; Jackiw, 1989; Jackobs and Rebbi, 1986; Krive and Rozhavskii, 1987; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b;

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0020-7748/02/0200-0251/0

Saint-James *et al.*, 1969), represent field-theoretical models which could be considered as effective theories of the Ginsburg–Landau-type for superconductivity (Banks and Lykken, 1990; Chen *et al.*, 1989; Fetter *et al.*, 1989). These models are in fact the relativistic generalizations of the well-known Ginsburg–Landau phenomenological field-theory models of superconductivity (Abrikosov, 1957a,b; Ginsburg and Landau, 1950). Some basics of the abelian Higgs models in the symmetry phase (Bogomol'nyi, 1976a,b; De Vega and Schaposnik, 1976; Friedberg and Lee, 1977a,b, 1978; Jackobs and Rebbi, 1986; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b) as well as in the broken symmetry phase (Boyanovsky, 1990), in one space, one time dimension, are recapitulated in the next section (Bogomol'nyi, 1976a,b; De Vega and Schaposnik, 1976; Friedberg and Lee, 1977a,b, 1978; Jackobs and Rebbi, 1986; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b).

Also, the quantization of field-theory models has always been a challenging problem. In a recent paper (Kulshreshtha, 2000b), the Hamiltonian (Dirac, 1950, 1964) and Becchi–Rouet–Stora–Tyutin (BRST) (Becchi *et al.*, 1974; Henneaux, 1985; Kulshreshtha, 1998, 2000a,b, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin) quantization of the abelian Higgs model in (1+1)-dimension have been studied in the broken symmetry phase (Boyanovsky, 1990) under some specific gauge choices in the usual instant-form, on the hyperplanes $x^0 =$ constant (Kulshreshtha, 2000b). In the present work, a consistent light-front Hamiltonian (Dirac, 1950, 1964) and BRST (Becchi *et al.*, 1974; Henneaux, 1985; Kulshreshtha, 1998, 2000a,b, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin) quantization of this theory in the broken symmetry phase (Boyanovsky, 1990) with some specific light-cone gauges (Dirac, 1949) is presented.

Further, in the usual Hamiltonian formulation of a gauge-invariant theory under some gauge-fixing conditions, one necessarily destroys the gauge invariance of the theory by fixing the gauge (which converts a set of first-class constraints into a set of second-class constraints, implying a breaking of gauge invariance under the gauge fixing). To achieve the quantization of a gauge-invariant theory such that the gauge invariance of the theory is maintained even under gauge fixing, one goes to a more generalized procedure called the BRST formulation (Becchi *et al.*, 1974; Henneaux, 1985; Kulshreshtha, 1998, 2000a,b, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin). In the BRST formulation of a gauge-invariant theory, the theory is rewritten as a quantum system that possesses a generalized gauge invariance called the BRST symmetry. For this, one enlarges the Hilbert space of the gauge-invariant theory and replaces the notion of the gauge transformation, which shifts operators by c-number functions, by a BRST transformation, which mixes operators having different statistics. In view of this, one introduces new anticommuting variables c and $\bar{c}$ called the Faddeev–Popov ghost and antighost fields, which are Grassmann numbers on the classical level and operators in the quantized theory, and a commuting variable b called the Nakanishi–Lautrup field (Becchi *et al.*, 1974; Henneaux, 1985; Kulshreshtha, 1998, 2000a,b, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin). In the BRST formulation, one thus embeds a gauge-invariant theory into a BRST-invariant system, and the quantum Hamiltonian of the system (which includes the gauge fixing contribution) commutes with the BRST charge operator Q as well as anti-BRST charge operator $\bar{Q}$. The new symmetry of the quantum system (the BRST symmetry) that replaces the gauge invariance is maintained (even under the gauge fixing) and hence projecting any state onto the sector of BRST-invariant and anti-BRST-invariant states yields a theory that is isomorphic to the original gauge-invariant theory.

The instant-from Hamiltonian and BRST formulations of the abelian Higgs model in the symmetry phase (Bogomol'nyi, 1976a,b; De Vega and Schaposnik, 1976; Friedberg and Lee, 1977a,b, 1978; Jackobs and Rebbi, 1986; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b) have been studied in Kulshreshtha (in press-a), and in the broken symmetry phase (where the phase of the complex matter field carries the charge degree of freedom of the complex matter field; Boyanovsky, 1990) in Kulshreshtha (2000b). In the present work the Hamiltonian and BRST formulations of the model are studied in the broken symmetry phase in the light-front frame, i.e. on the hyperplanes $\sqrt{2}x^+ = (x^0 + x^1) =$ constant (Dirac, 1949). After a brief recapitulation of the basics of the abelian Higgs model (in the symmetry phase as well as in the broken symmetry phase) in the next section, its Hamiltonian formulation in the broken symmetry phase in the light-front frame is considered in Section 3 and its corresponding light-front BRST formulation (also in the broken symmetry phase) is studied in Section 4.

2. SOME BASICS OF THE MODEL: A RECAPITULATION

2.1. Model in the Symmetry Phase

The two-dimensional abelian Higgs model in the symmetry phase is defined by the action (Bogomol'nyi, 1976a,b; De Vega and Schaposnik, 1976; Friedberg and Lee, 1977a,b, 1978; Jackobs and Rebbi, 1986; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b):

$$S = \int \mathscr{L}(\Phi, \Phi^*, A^\mu)\, d^2x \tag{2.1a}$$

$$\mathscr{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + (\tilde{D}_\mu \Phi^*)(D^\mu \Phi) - V(|\Phi|^2) \tag{2.1b}$$

$$V(|\Phi|^2) = \alpha_0 + \alpha_2 |\Phi|^2 + \alpha_4 |\Phi|^4 \tag{2.1c}$$

$$= \lambda\big(|\Phi|^2 - \Phi_0^2\big)^2, \quad \Phi_0 \neq 0 \tag{2.1d}$$

$$D_\mu = (\partial_\mu + ieA_\mu), \qquad \tilde{D}_\mu = (\partial_\mu - ieA_\mu) \tag{2.1e}$$

$$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) \tag{2.1f}$$

$$g^{\mu\nu} := \mathrm{diag}\,(+1, -1), \quad \mu, \nu = 0, 1 \tag{2.1g}$$

The model is well known to possess stable, time-independent (i.e., static), classical solutions called the topological solitons of the vortex type (Banerjee *et al.*, 1995, 1997; Bogomol'nyi, 1976a,b; De Vega and Schaposnik, 1976; Dunne and Trugenberger, 1991; Forte, 1992; Friedberg and Lee, 1977a,b, 1978; Jackiw, 1989; Jackobs and Rebbi, 1986; Krive and Rozhavskii, 1987; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b; Saint-James *et al.*, 1969).

In a quantum theory of the kind that we are considering here, for a specific form of the Higgs potential which admits static solutions, in general, one could have *two* degenerate minima—a symmetry breaking minimum and a symmetry preserving minimum—and correspondingly the theory could have two types of classical solutions—the topological vortices with quantized magnetic flux as we have in the Ginsburg–Landau model, where it is possible to define a conserved topological current and a corresponding topological charge which is quantized and is related to the topological quantum number called the winding number and the other type of classical solutions are the nontopological solitons with nonvanishing but not necessarily quantized magnetic flux (Banerjee *et al.*, 1995, 1997; Bogomol'nyi, 1976a,b; De Vega and Schaposnik, 1976; Dunne and Trugenberger, 1991; Friedberg and Lee, 1977a,b, 1978; Jackobs and Rebbi, 1986; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b). The main new result of such studies is the identification of the Ginsburg–Landau theory with the static solution of the Higgs type of Lagrangian (Abrikosov, 1957a,b; Banerjee *et al.*, 1995, 1997; Banks and Lykken, 1990; Bogomol'nyi, 1976a,b; Chen *et al.*, 1989; De Vega and Schaposnik, 1976; Dunne and Trugenberger, 1991; Fetter *et al.*, 1989; Forte, 1992; Friedberg and Lee, 1977a,b, 1978; Ginsburg and Landau, 1950; Jackiw, 1989; Jackobs and Rebbi, 1986; Krive and Rozhavskii, 1987; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b; Saint-James *et al.*, 1969).

Further, in the present model, considered with a Higgs potential in the form of a double well potential with $\Phi_0 \neq 0$, the spontaneous symmetry breaking takes place owing to the noninvariance of the lowest (ground) state of the system (because $\Phi_0 \neq 0$) under the operation of the local $U(1)$ symmetry. Also the symmetry that is broken is still a symmetry of the system and it is manifested in a manner other than the invariance of the lowest or ground state (Φ_0) of the system. However, no Goldstone boson occurs here and instead the gauge field acquires a mass through some kind of a Higgs mechanism and the symmetry is manifested in the Higgs mode.

In general, one can keep here the Higgs potential rather general, i.e., without making any specific choice for the parameters of the potential except that they are chosen such that the potential remains a double well potential with $\Phi_0 \neq 0$. For further details we refer to the work of Banerjee *et al.* (1995, 1997), Bogomol'nyi (1976a,b), De Vega and Schaposnik (1976), Dunne and Trugenberger (1991), Friedberg and Lee (1977a,b, 1978), Jackobs and Rebbi (1986), Kulshreshtha (in press-b), Lee *et al.* (1991), Lee and Nam (1991), Nielsen and Olesen (1973a,b), and references therein. The instant-form model in the symmetry phase has been studied in Kulshreshtha (in press-a). This theory considered in the symmetry phase in the instant-form is seen to possess a set of two first-class constraints (Kulshreshtha, in press-a):

$$\chi_1 = \Pi_0 \approx 0 \tag{2.2a}$$

$$\chi_2 = [\partial_1 E - ie(\Pi^* \Phi^* - \Pi \Phi)] \approx 0 \tag{2.2b}$$

where χ_1 is a primary constraint and χ_2 is the secondary Gauss-law constraint. The theory is accordingly seen to possess a local vector gauge symmetry and remains invariant under the local vector gauge transformations (Kulshreshtha, in press-a):

$$\delta\Phi = i\beta\Phi, \qquad \delta\Phi^* = -i\beta\Phi^*, \tag{2.3a}$$

$$\delta A_0 = -\partial_0\beta, \qquad \delta A_1 = -\partial_1\beta, \qquad \delta\Pi_0 = 0, \qquad \partial E = 0 \tag{2.3b}$$

$$\delta\Pi = -e\beta A_0\Phi^* - i\beta\partial_0\Phi^* + i(e-1)\Phi^*\partial_0\beta \tag{2.3c}$$

$$\delta\Pi^* = -e\beta A_0\Phi + i\beta\partial_0\Phi - i(e-1)\Phi\partial_0\beta \tag{2.3d}$$

where the gauge parameter $\beta = \beta(x^\mu)$ is a function of its arguments. Also, the divergence of the vector gauge current density for the theory is seen to vanish explicitly (Kulshreshtha, in press-a), implying that the theory possesses at the classical level a local vector gauge symmetry, which is seen to be consistent with the above results (Kulshreshtha, in press-a). The Hamiltonian and BRST formulations of this model in the symmetry phase in the instant-form have been studied in details in Kulshreshtha (in press-a) under some specific gauge choices (Kulshreshtha, in press-a) and for further details we refer to the work of Kulshreshtha (in press-a).

The Hamiltonian and BRST formulations of this theory in the broken symmetry phase in the usual instant-form have been studied in Kulshreshtha (2000b). In the present work, the theory is studied in the broken symmetry phase in the light-front frame (Dirac, 1949) on the hyperplanes $\sqrt{2}x^+ \equiv \sqrt{2}t = (x^0 + x^1) =$ constant (Dirac, 1949; Kulshreshtha and Kulshreshtha, 2000; Srivastava, 1998, 1999).

2.2. Model in the Broken Symmetry Phase

In the present work we study the abelian Higgs model in the broken symmetry phase (Boyanovsky, 1990) of the complex matter field Φ [$\equiv\Phi(x^\mu)$] on the light-front (Dirac, 1949). For this purpose, for the complex matter field Φ we take (Boyanovsky, 1990)

$$\Phi = \Phi_0 \exp[i\phi], \quad \Phi_0 \neq 0 \tag{2.4}$$

Here ϕ [$\equiv\phi(x^\mu)$] is the phase of the complex scalar field Φ. The action of the theory (Bogomol'nyi, 1976a,b; De Vega and Schaposnik, 1976; Friedberg and Lee, 1977a,b, 1978; Jackobs and Rebbi, 1986; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b) in the broken symmetry phase (Boyanovsky, 1990) then becomes

$$S = \int \mathscr{L}\, d^2x \tag{2.5a}$$

$$\mathscr{L} := \frac{-1}{4e^2} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\phi + eA_\mu)(\partial^\mu\phi + eA^\mu) \tag{2.5b}$$

It is important to notice here that the vector gauge boson A^μ becomes massive in the broken symmetry phase. This mass generation of the vector gauge boson takes place perhaps through a mechanism similar to the Higgs mechanism (Boyanovsky, 1990). The phase ϕ carries the charge degree of freedom of Φ and is in fact akin to the Goldstone boson and is to be treated as a dynamical field (Boyanovsky, 1990). Also the ground state in the broken symmetry phase is not rotational invariant. Thus the theory considered in the broken symmetry phase can be thought of as a Higgs Lagrangian where the Higgs potential has been set to zero by freezing the complex matter field Φ at the degenerate minima of the potential. Such studies of the theory in the broken symmetry (superfluid) state could be relevant for the effective theories in the condensed matter as the action of the theory describes the low-lying excitations in the broken symmetry phase (Boyanovsky, 1990). In the present work we study the Hamiltonian and BRST formulations of the theory described by the action (2.5) in the light-front frame.

2.2.1 The Instant-Form Theory

This theory considered in the broken symmetry phase defined by (2.5) when considered in the instant-form is seen to possess a set of two first-class constraints (Kulshreshtha, 2000b):

$$\rho_1 = \Pi_0 \approx 0 \tag{2.6a}$$

$$\rho_2 = [\partial_1 E + e\pi] \approx 0 \tag{2.6b}$$

where ρ_1 is a primary constraint and ρ_2 is the secondary Gauss-law constraint. The theory is accordingly seen to possess a local vector gauge symmetry and remains invariant under the local vector gauge transformations (Kulshreshtha, 2000b):

$$\delta\phi = e\beta, \qquad \delta A_0 = -\partial_0\beta, \qquad \delta A_1 = -\partial_1\beta \tag{2.7a}$$

$$\delta\pi = \delta\Pi_0 = \delta E = 0 \tag{2.7b}$$

where the gauge parameter $\beta = \beta(x^\mu)$ is a function of its arguments. Also, the divergence of the vector gauge current density for the theory is seen to vanish explicitly (Kulshreshtha, 2000b), implying that the theory possesses at the classical level a local vector gauge symmetry, which is seen to be consistent with the above results (Kulshreshtha, 2000b). The Hamiltonian and BRST formulations of this model in the symmetry phase in the instant-form have been studied in details in Kulshreshtha (2000b) under some specific gauge choices, and for further details we refer to the work of Kulshreshtha (2000b).

In the following section we consider the Hamiltonian and BRST formulations of the light-front theory in the broken symmetry phase.

3. HAMILTONIAN FORMULATION OF THE LIGHT-FRONT THEORY IN THE BROKEN SYMMETRY PHASE

For considering the Hamiltonian formulation of the model in the broken symmetry phase in the light-front frame (i.e., on the hyperplanes $\sqrt{2}x^+ = \sqrt{2}t = (x^0 + x^1) = \text{constant}$) (Dirac, 1949), we express the action of the theory (2.5) in the light-front frame (Dirac, 1949), which in (1+1)-dimensions reads as (Banerjee *et al.*, 1995, 1997; Bogomol'nyi, 1976a,b; De Vega and Schaposnik, 1976; Dunne and Trugenberger, 1991; Friedberg and Lee, 1977a,b, 1978; Jackobs and Rebbi, 1986; Kulshreshtha, in press-a; Lee *et al.*, 1991; Lee and Nam, 1991; Nielsen and Olesen, 1973a,b)

$$S = \int \mathscr{L}\, dx^+\, dx^- \tag{3.1a}$$

$$\begin{aligned}\mathscr{L} = {}& \frac{1}{2e^2}[\partial_+ A^+ - \partial_- A^-]^2 + (\partial_+\phi)(\partial_-\phi) \\ & + e[A^-(\partial_-\phi) + A^+(\partial_+\phi)] + e^2 A^+ A^-\end{aligned} \tag{3.1b}$$

In the following, we would consider the Hamiltonian formulation of the theory described by the action (3.1). The Euler–Lagrange field equations of motion of the theory obtained from (3.1) are

$$-2\partial_-\partial_+\phi - e(\partial_+ A^+ + \partial_- A^-) = 0 \tag{3.2a}$$

$$e(\partial_+\phi) + e^2 A^- - \frac{1}{e^2}\partial_+(\partial_+ A^+ - \partial_- A^-) = 0 \tag{3.2b}$$

$$e(\partial_-\phi) + e^2 A^+ + \frac{1}{e^2}\partial_-(\partial_+ A^+ - \partial_- A^-) = 0 \tag{3.2c}$$

The canonical momenta obtained from (3.1) are

$$\pi := \frac{\partial \mathscr{L}}{\partial(\partial_+\phi)} = (\partial_-\phi + eA^+) \tag{3.3a}$$

$$\Pi^+ := \frac{\partial \mathscr{L}}{\partial(\partial_+ A^-)} = 0 \tag{3.3b}$$

$$\Pi^- := \frac{\partial \mathscr{L}}{\partial(\partial_+ A^+)} = \frac{1}{e^2}(\partial_+ A^+ - \partial_- A^-) \tag{3.3c}$$

Here π, Π^+, and Π^- are the momenta canonically conjugate, respectively, to ϕ, A^-, and A^+. Equations (3.3) imply that the theory possesses two primary constraints:

$$\chi_1 = \Pi^+ \approx 0, \qquad \chi_2[\pi - \partial_-\phi - eA^+] \approx 0 \tag{3.4}$$

Here the symbol $\approx$ denotes the *weak* equality in the sense of Dirac, and it implies that the left-hand side of the equation vanishes strongly only on the hypersurface or the reduced surface of the constraints of the theory and it is nonvanishing everywhere else in the rest of the phase space of the theory (Dirac, 1950, 1964). The canonical Hamiltonian density corresponding to $\mathscr{L}$ (3.1) is

$$\mathcal{H}_c := \pi(\partial_+\phi) + \Pi^+(\partial_+ A^-) + \Pi^-(\partial_+ A^+) - \mathscr{L} \tag{3.5a}$$

$$= \frac{1}{2}e^2(\Pi^-)^2 + \Pi^-(\partial_- A^-) - eA^-(\partial_-\phi) - e^2 A^+ A^- \tag{3.5b}$$

After including the primary constant χ_1 in the canonical Hamiltonian density $\mathcal{H}_c$ with the help of the Lagrange multiplier field u, the total Hamiltonian density $\mathcal{H}_T$ could be written as

$$\mathcal{H}_T = \frac{1}{2}e^2(\Pi^-)^2 + \Pi^-(\partial_- A^-) - eA^-(\partial_-\phi) - e^2 A^+ A^- + \Pi^+ u + (\pi - \partial_-\phi - eA^+)v \tag{3.6}$$

The Hamilton's equations obtained from the total Hamiltonian

$$H_T = \int \mathcal{H}_T \, dx^- \tag{3.7}$$

are

$$\partial_+\phi = \frac{\partial H_T}{\partial \pi} = v \tag{3.8a}$$

$$-\partial_+\pi = \frac{\partial H_T}{\partial \phi} = \partial_- v + e\,\partial_- A^- \tag{3.8b}$$

$$\partial_+ A^- = \frac{\partial H_\mathrm{T}}{\partial \Pi^+} = u \tag{3.8c}$$

$$-\partial_+ \Pi^+ = \frac{\partial H_\mathrm{T}}{\partial A^-} = -\partial_- \Pi^- - e\,\partial_- \phi - e^2 A^+ \tag{3.8d}$$

$$\partial_+ A^+ = \frac{\partial H_\mathrm{T}}{\partial \Pi^-} = e^2 \Pi^- + \partial_- A^- \tag{3.8e}$$

$$-\partial_+ \Pi^- = \frac{\partial H_\mathrm{T}}{\partial A^+} = -e^2 A^- - ev \tag{3.8f}$$

$$\partial_+ u = \frac{\partial H_\mathrm{T}}{\partial \Pi_u} = 0 \tag{3.8g}$$

$$-\partial_+ \Pi_u = \frac{\partial H_\mathrm{T}}{\partial u} = \Pi^+ \tag{3.8h}$$

$$\partial_+ v = \frac{\partial H_\mathrm{T}}{\partial \Pi_v} = 0 \tag{3.8i}$$

$$-\partial_+ \Pi_v = \frac{\partial H_\mathrm{T}}{\partial v} = \Pi - \partial_- \phi - eA^+ \tag{3.8j}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. The Lagrange multiplier fields u and v as well as their canonically conjugate momenta Π_u and Π_v are to be treated henceforth as the dynamical fields like all other field variables of the theory. Their dynamics is given by the Hamilton's equations (3.8g)–(3.8j). For the Poisson bracket $\{\ ,\ \}_\mathrm{p}$ of two functions A and B, we choose the following convention:

$$\{A(x), B(y)\}_\mathrm{p} := \int dz \sum_\alpha \left[\frac{\partial A(x)}{\partial q_\alpha(z)} \frac{\partial B(y)}{\partial p_\alpha(z)} - \frac{\partial A(x)}{\partial p_\alpha(z)} \frac{\partial B(y)}{\partial q_\alpha(z)} \right] \tag{3.9}$$

Demanding that the primary constraint χ_1 be preserved in the course of time, one obtains the secondary Gauss-law constraint of the theory as

$$\chi_3 := \{\chi_1, \mathcal{H}_\mathrm{T}\}_\mathrm{p} = [\partial_- \Pi^- + e\,\partial_- \phi + e^2 A^+] \approx 0 \tag{3.10}$$

The preservation of χ_2 for all times does not give rise to any further constraints. The theory is thus seen to possess only three constraints χ_1, χ_2, and χ_3:

$$\chi_1 = \Pi^+ \approx 0 \tag{3.11a}$$

$$\chi_2 = [\pi - \partial_- \phi - eA^+] \approx 0 \tag{3.11b}$$

$$\chi_3 = [\partial_- \Pi^- + e\,\partial_- \phi + e^2 A^+] \approx 0 \tag{3.11c}$$

where χ_1 and χ_2 are primary constraints and χ_3 is a secondary constraint.

Further, the matrix of the Poisson brackets of the constraints χ_i is seen to be a singular matrix, implying that the set of constraints χ_i is first-class and that the theory described by (3.1) is a gauge-invariant theory. The action of the theory $S(3.1)$ is in fact seen to be invariant under the local vector gauge transformations:

$$\delta\phi = e\beta, \quad \delta A^+ = -\partial_-\beta, \qquad \delta A^- = -\partial_+\beta, \qquad \delta u = -\partial_+\partial_+\beta \tag{3.12a}$$

$$\delta v = e\partial_+\beta, \quad \delta\pi = \delta\Pi^+ = \delta\Pi^- = \delta\Pi_v = \delta\Pi_u = 0 \tag{3.12b}$$

where $\beta \equiv \beta(x^+, x^-)$ is an arbitrary function of its arguments. The generator of the local vector gauge transformations is the charge operator of the theory:

$$J^+ = \int j^+ \, dx^- = \int dx^- \left[e\beta(\partial_-\phi + eA^+) - \frac{1}{e^2}(\partial_-\beta)(\partial_+ A^+ - \partial_- A^-) \right] \tag{3.13}$$

The current operator of the theory is

$$J^- = \int j^- \, dx^- = \int dx^- \left[e\beta(\partial_+\phi + eA^-) + \frac{1}{e^2}(\partial_+\beta)(\partial_+ A^+ - \partial_- A^-) \right] \tag{3.14}$$

The divergence of the vector gauge current density, namely $\partial_\mu j^\mu$, is therefore seen to vanish so that

$$\partial_\mu j^\mu = \partial_+ j^+ + \partial_- j^- = 0 \tag{3.15}$$

implying that the theory possesses (at the classical level) a local vector gauge symmetry.

In order to quantize the theory using Dirac's procedure we convert the set of first-class constraints of the theory χ_i into a set of second-class constraints, by imposing, arbitrarily, some additional constraints on the system called gauge-fixing conditions or the gauge constraints. For this purpose, for the present theory, we could choose, for example, the set of gauge-fixing conditions: (A) $\rho_1 = A^+ = 0$ and $\rho_2 = A^- = 0$; and (B) $\psi_1 = \partial_- A^+ = 0$ and $\psi_2 = \partial_- A^- = 0$. Corresponding to this choice of the gauge-fixing conditions, we have the following two sets of constraints under which the quantization of the theory could be studied:

(A)

$$\xi_1 = \chi_1 = \Pi^+ \approx 0 \tag{3.16a}$$

$$\xi_2 = \chi_2 = [\pi - \partial_-\phi - eA^+] \approx 0 \tag{3.16b}$$

$$\xi_3 = \chi_3 = [\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+] \approx 0 \tag{3.16c}$$

$$\xi_4 = \rho_1 = A^+ \approx 0 \tag{3.16d}$$

$$\xi_5 = \rho_2 = A^- \approx 0 \tag{3.16e}$$

and

(B)

$$\eta_1 = \chi_1 = \Pi^+ \approx 0 \tag{3.17a}$$

$$\eta_2 = \chi_2 = [\pi - \partial_- \phi - eA^+] \approx 0 \tag{3.17b}$$

$$\eta_3 = \chi_3 = [\partial_- \Pi^- + e\,\partial_- \phi + e^2 A^+] \approx 0 \tag{3.17c}$$

$$\eta_4 = \psi_1 = \partial_- A^+ \approx 0 \tag{3.17d}$$

$$\eta_5 = \psi_2 = \partial_- A^- \approx 0 \tag{3.17e}$$

The matrices of the Poisson brackets among the set of constraints ξ_i and η_i are now seen to be nonsingular (and therefore invertible) and are omitted here for the sake of brevity. It is important to note here that the set of gauge-fixing conditions chosen here in either (3.16) or (3.17) are not only consistent with the Dirac procedure (Dirac, 1950, 1964), but are also of physical importance, in the sense that the gauge choice $A^+ = 0$, or equivalently $\partial_- A^+ = 0$, represents the temporal or the time-axial gauge and the gauge choice $A^- = 0$, or equivalently $\partial_- A^- = 0$, represents the so-called Coulomb gauge.

The Dirac bracket $\{\ ,\ \}_{\mathrm{D}}$ of the two functions A and B is defined as (Dirac, 1950, 1964)

$$\{A, B\}_{\mathrm{D}} = \{A, B\}_{\mathrm{p}} - \int\!\!\int dw\, dz \sum_{\alpha,\beta} \left[\{A, \Gamma_\alpha(w)\}_{\mathrm{p}} \left[\Delta^{-1}_{\alpha\beta}(w, z)\right] \{\Gamma_\beta(z), B\}_{\mathrm{p}}\right] \tag{3.18}$$

where Γ_i are the constraints of the theory and $\Delta_{\alpha\beta}(w, z)\ [=\{\Gamma_\alpha(w), \Gamma_\beta(z)\}_{\mathrm{p}}]$ is the matrix of the Poisson brackets of the constraints Γ_i. The transition to quantum theory is made by the replacement of the Dirac brackets by the operator commutation relations according to

$$\{A, B\}_{\mathrm{D}} \longrightarrow (-i)[A, B], \quad i = \sqrt{-1} \tag{3.19}$$

Finally, the nonvanishing equal-time commutators of the theory in Case A, i.e., in the gauge $A^+ = 0$ and $A^- = 0$, are obtained as (Kulshreshtha, 1998, 2000, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999)

$$[\phi(x), \phi(y)] = -\frac{1}{4} i \epsilon(x^- - y^-) \tag{3.20a}$$

$$[\phi(x), \pi(y)] = \frac{3}{2} i \delta(x^- - y^-) \tag{3.20b}$$

$$[\phi(x), \Pi^-(y)] = -\frac{3}{4} i e \epsilon(x^- - y^-) \tag{3.20c}$$

$$[\pi(x), \pi(y)] = -\frac{1}{2} i\, \partial_- \delta(x^- - y^-) \tag{3.20d}$$

$$[\pi(x), \Pi^-(y)] = -\frac{1}{2} i e \delta(x^- - y^-) \tag{3.20e}$$

$$[\Pi^-(x), \Pi^-(y)] = -\frac{1}{4} i e^2 \epsilon(x^- - y^-) \tag{3.20f}$$

where $\epsilon(x - y)$ is a step function defined as

$$\epsilon(x-y) := \begin{cases} +1, & (x-y) > 0 \\ -1, & (x-y) < 0 \end{cases} \tag{3.21}$$

The nonvanishing equal-time commutators of the theory in Case B, i.e., in the gauge $\partial_- A^+ = 0$ and $\partial_- A^- = 0$, are seen to be identical with those of Case A as they should, and are given by (3.20). This is not surprising in view of the fact that the gauges $A^+ = 0$ and $\partial_- A^+ = 0$, as well as $A^- = 0$ and $\partial_- A^- = 0$, conceptually mean the same.

For later use, for considering the BRST formulation of the theory we convert the total Hamiltonian density into the first-order Lagrangian density $\mathscr{L}_{10}$:

$$\begin{aligned} \mathscr{L}_{10} &:= \pi(\partial_+\phi) + \Pi^+(\partial_+ A^-) + \Pi^-(\partial_+ A^+) + \Pi_u(\partial_+ u) + \Pi_v(\partial_+ v) - \mathcal{H}_\mathrm{T} \\ &= \Pi^-(\partial_+ A^+) - \Pi^-(\partial_- A^-) - \frac{1}{2} e^2 (\Pi^-)^2 + eA^-(\partial_-\phi) + e^2 A^+ A^- \\ &\quad + (\partial_+\phi)(\partial_-\phi + eA^+) + \Pi_u(\partial_+ u) + \Pi_v(\partial_+ v) \end{aligned} \tag{3.22}$$

In the above equation, the term $\Pi^+(\partial_+ A^- - u)$ drops out in view of the Hamilton's equations. The appearance of the field variables u, v, Π_u, and Π_v in the above equation and hereafter, however, does not pose any problems because the variables u, v, Π_u, and Π_v in our treatment are dynamical field variables whose dynamics is governed by the Hamilton's equations (3.8g)–(3.8j).

4. BRST FORMULATION OF THE LIGHT-FRONT THEORY IN THE BROKEN SYMMETRY PHASE

4.1. The BRST Invariance

For the BRST formulation of the model, we rewrite our theory under consideration as a quantum system that possesses the generalized gauge invariance called BRST symmetry. For this, we first enlarge the Hilbert space of our gauge-invariant theory and replace the notion of gauge transformation, which shifts operators by c-number functions, by a BRST transformation, which mixes operators with Bose and Fermi statistics. We then introduce new anticommuting variables c and $\bar{c}$ (Grassman numbers on the classical level and operators in the quantized theory) and a commuting variable b such that (Becchi, *et al.*, 1974; Henneaux, 1985; Kulshreshtha, 1998, 2000a, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin):

$$\hat{\delta}\phi = ec, \qquad \hat{\delta}A^+ = -\partial_- c, \qquad \hat{\delta}A^- = -\partial_+ c \tag{4.1a}$$

$$\hat{\delta}\pi = \hat{\delta}\Pi^{+} = \hat{\delta}\Pi^{-} = \hat{\delta}\Pi_{u} = \hat{\delta}\Pi_{v} = 0, \quad \hat{\delta}u = -\partial_{+}\partial_{+}c, \quad \hat{\delta}v = e\,\partial_{+}c \tag{4.1b}$$

$$\hat{\delta}c = 0, \qquad \hat{\delta}\bar{c} = b, \qquad \hat{\delta}b = 0 \tag{4.1c}$$

with the property $\hat{\delta}^2 = 0$. We now define a BRST-invariant function of the dynamical variables to be a function $f(\pi, \Pi^{+}, \Pi^{-}, \Pi_{u}, \Pi_{v}, p_{\mathrm{b}}, \Pi_{\mathrm{c}}, \Pi_{\bar{\mathrm{c}}}, \phi, A^{+}, A^{-}, u, v, b, c, \bar{c})$ such that $\hat{\delta} f = 0$.

4.2. Gauge Fixing in the BRST Formalism

Performing gauge fixing in the BRST formalism implies adding to the first-order Lagrangian density $\mathscr{L}_{10}$, a trivial BRST-invariant function (Becchi *et al.*, 1974; Henneaux, 1985; Kulshreshtha, 1998, 2000a,b, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press-b; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin). We could thus write, for example (where u, v, Π_u, and Π_v are to be treated as dynamical field variables),

$$\begin{aligned}\mathscr{L}_{\mathrm{BRST}} = {} & \Pi^{-}(\partial_{+}A^{+}) - \Pi^{-}(\partial_{-}A^{-}) - \frac{1}{2}e^{2}(\Pi^{-})^{2} + eA^{-}(\partial_{-}\phi) + e^{2}A^{+}A^{-} \\ & + (\partial_{+}\phi)(\partial_{-}\phi + eA^{+}) + \Pi_{u}(\partial_{+}u) + \Pi_{v}(\partial_{+}v) \\ & + \hat{\delta}\left[\bar{c}\left(-\partial_{+}A^{-} + \frac{1}{e}\phi - \frac{1}{2}b\right)\right]\end{aligned} \tag{4.2}$$

The last term in the above equation is the extra BRST-invariant, gauge-fixing term. After one integration by parts, the above equation could now be written as

$$\begin{aligned}\mathscr{L}_{\mathrm{BRST}} = {} & \Pi^{-}(\partial_{+}A^{+}) - \Pi^{-}(\partial_{-}A^{-}) - \frac{1}{2}e^{2}(\Pi^{-})^{2} + eA^{-}(\partial_{-}\phi) + e^{2}A^{+}A^{-} \\ & + (\partial_{+}\phi)(\partial_{-}\phi + eA^{+}) + \Pi_{u}(\partial_{+}u) + \Pi_{v}(\partial_{+}v) \\ & + b\left(-\partial_{+}A^{-} + \frac{1}{e}\phi\right) - \frac{1}{2}b^{2} + (\partial_{+}\bar{c})(\partial_{+}c) - \bar{c}c\end{aligned} \tag{4.3}$$

Proceeding classically, the Euler–Lagrange equation for b reads

$$-b = \left(\partial_{+}A^{-} - \frac{1}{e}\phi\right) \tag{4.4}$$

The requirement $\hat{\delta}b = 0$ then implies

$$-\hat{\delta}b = \left[\hat{\delta}(\partial_{+}A^{-}) - \frac{1}{e}\hat{\delta}\phi\right] = 0 \tag{4.5}$$

which in turn implies [by setting $\hat{\delta}b = 0$]

$$[-\partial_{+}\partial_{+}c] = c. \tag{4.6}$$

The above equation is also an Euler–Lagrange equation obtained by the variation of $\mathscr{L}_{\text{BRST}}$ with respect to $\bar{c}$. In introducing momenta one has to be careful in defining those for the fermionic variables. We thus define the bosonic momenta in the usual manner so that

$$\Pi^+ := \frac{\partial}{\partial(\partial_+ A^-)} \mathscr{L}_{\text{BRST}} = -b \tag{4.7}$$

but for the fermionic momenta with directional derivatives we set

$$\Pi_c := \mathscr{L}_{\text{BRST}} \frac{\overleftarrow{\partial}}{\partial(\partial_+ c)} = \partial_+ \bar{c}, \qquad \Pi_{\bar{c}} := \frac{\overrightarrow{\partial}}{\partial(\partial_+ \bar{c})} \mathscr{L}_{\text{BRST}} = \partial_+ c \tag{4.8}$$

implying that the variable canonically conjugate to c is $(\partial_+ \bar{c})$ and the variable conjugate to $\bar{c}$ is $(\partial_+ c)$. For writing the Hamiltonian density from the Lagrangian density in the usual manner we remember that the former has to be Hermitian so that

$$\begin{aligned}
\mathcal{H}_{\text{BRST}} &= \pi(\partial_+\phi) + \Pi^+(\partial_+ A^-) + \Pi^-(\partial_+ A^+) + \Pi_u(\partial_+ u) + \Pi_v(\partial_+ v) \\
&\quad + \Pi_c(\partial_+ c) + \Pi_{\bar{c}}(\partial_+ \bar{c}) - \mathscr{L}_{\text{BRST}} \\
&= \frac{1}{2} e^2 (\Pi^-)^2 + \Pi^-(\partial_- A^-) - eA^-(\partial_-\phi) - e^2 A^+ A^- \\
&\quad + \Pi^+ \left(\frac{1}{e}\phi\right) + \frac{1}{2}(\Pi^+)^2 + \Pi_c \Pi_{\bar{c}} + \bar{c}c
\end{aligned} \tag{4.9}$$

We can check the consistency of (4.8) and (4.9) by looking at Hamilton's equations for the fermionic variables, i.e.,

$$\partial_+ c = \frac{\overrightarrow{\partial}}{\partial \Pi_c} \mathcal{H}_{\text{BRST}}, \qquad \partial_+ \bar{c} = \mathcal{H}_{\text{BRST}} \frac{\overleftarrow{\partial}}{\partial \Pi_{\bar{c}}} \tag{4.10}$$

Thus we see that

$$\partial_+ c = \frac{\overrightarrow{\partial}}{\partial \Pi_c} \mathcal{H}_{\text{BRST}} = \Pi_{\bar{c}}, \qquad \partial_+ \bar{c} = \mathcal{H}_{\text{BRST}} \frac{\overleftarrow{\partial}}{\partial \Pi_{\bar{c}}} = \Pi_c \tag{4.11}$$

is in agreement with (4.8). For the operators c, $\bar{c}$, $\partial_+ c$, and $\partial_+ \bar{c}$, one needs to satisfy the anticommutation relations of $\partial_+ c$ with $\bar{c}$ or of $\partial_+ \bar{c}$ with c, but not of c with $\bar{c}$. In general, c and $\bar{c}$ are independent canonical variables and one assumes that

$$\{\Pi_c, \Pi_{\bar{c}}\} = \{\bar{c}, c\} = 0, \qquad \partial_+\{\bar{c}, c\} = 0 \tag{4.12a}$$

$$\{\partial_+ \bar{c}, c\} = (-1)\{\partial_+ c, \bar{c}\} \tag{4.12b}$$

where { , } means an anticommutator. We thus see that the anticommutators in (4.12b) are nontrivial and need to be fixed. In order to fix these, we require that c satisfy the Heisenberg equation (Becchi *et al.*, 1974; Henneaux, 1985;

Kulshreshtha, 1998, 2000a, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin):

$$[c, \mathcal{H}_{\mathrm{BRST}}] = i\,\partial_+ c \tag{4.13}$$

and using the property $c^2 = \bar{c}^2 = 0$ one obtains

$$[c, \mathcal{H}_{\mathrm{BRST}}] = \{\partial_+\bar{c}, c\}\,\partial_+ c \tag{4.14}$$

Equations (4.12)–(4.14) then imply

$$\{\partial_+\bar{c}, c\} = (-1)\{\partial_+ c, \bar{c}\} = i \tag{4.15}$$

The minus sign in the above equation is nontrivial and implies the existence of states with negative norm in the space of state vectors of the theory (Becchi *et al.*, 1974; Bogomol'nyi, 1976; De Vega and Schaposnik, 1976; Dirac, 1950, 1964; Henneaux, 1985; Jackobs and Rebbi, 1986; Nemeschansky *et al.*, 1988; Nielsen and Olesen, 1973a,b; Tyutin, xxxx).

4.3. The BRST Charge Operator

The BRST charge operator Q is the generator of the BRST transformations (4.1). It is nilpotent and satisfies $Q^2 = 0$. It mixes operators which satisfy Bose and Fermi statistics. According to its conventional definition, its commutators with Bose operators and its anticommutators with Fermi operators for the present theory satisfy

$$[\phi, Q] = (\partial_+ c), \quad [A^+, Q] = -(\partial_- c), \quad [A^-, Q] = (\partial_+ c) \tag{4.16a}$$

$$[\pi, Q] = e\,(\partial_- c) + \partial_-\partial_+ c, \quad [\Pi^-, Q] = e^2 c + e(\partial_+ c) \tag{4.16b}$$

$$\begin{aligned} \{\partial_+\bar{c}, Q\} &= -\partial_-\Pi^- - e\,\partial_-\phi - e^2 A^+, \\ \{\bar{c}, Q\} &= (-1)[\pi + \Pi^+ - \partial_-\phi - eA^+] \end{aligned} \tag{4.16c}$$

All other commutators and anticommutators involving Q vanish. In view of (4.16) the BRST charge operator of the present theory can be written as

$$Q = \int dx^-\,[ic[\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+] - i(\partial_+ c)[\pi + \Pi^+ - \partial_-\phi - eA^+]] \tag{4.17}$$

This equation implies that the set of states satisfying the conditions

$$\Pi^+|\psi\rangle = 0 \tag{4.18a}$$

$$[\pi - \partial_-\phi - eA^+]|\psi\rangle = 0 \tag{4.18b}$$

$$[\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+]|\psi\rangle = 0 \tag{4.18c}$$

belongs to the dynamically stable subspace of states $|\psi\rangle$ satisfying $Q|\psi\rangle = 0$, i.e., it belongs to the set of BRST-invariant states.

In order to understand the condition needed for recovering the physical states of the theory we rewrite the operators c and $\bar{c}$ in terms of fermionic annihilation and creation operators. For this purpose we consider (4.6). The solution of Eq. (4.6) (with the light-cone time variable x^+ defined as equal to t) gives the Heisenberg operator $c(t)$ (and correspondingly $\bar{c}(t)$) as

$$c(t) = e^{it}B + e^{-it}D, \qquad \bar{c}(t) = e^{-it}B^\dagger + e^{it}D^\dagger \tag{4.19}$$

which at light-cone time $t(\equiv x^+) = 0$ imply

$$c \equiv c(0) = B + D, \qquad \bar{c} \equiv \bar{c}(0) = B^\dagger + D^\dagger \tag{4.20a}$$

$$\partial_+ c \equiv \partial_+ c(0) = i(B - D), \qquad \partial_+ \bar{c} \equiv \partial_+ \bar{c}(0) = -i(B^\dagger - D^\dagger) \tag{4.20b}$$

By imposing the conditions

$$c^2 = \bar{c}^2 = \{\bar{c}, c\} = \{\partial_+\bar{c}, \partial_+ c\} = 0 \tag{4.21a}$$

$$\{\partial_+\bar{c}, c\} = i = (-1)\{\partial_+ c, \bar{c}\} \tag{4.21b}$$

we now obtain the equations

$$B^2 + \{B, D\} + D^2 = B^{\dagger 2} + \{B^\dagger, D^\dagger\} + D^{\dagger 2} = 0 \tag{4.22a}$$

$$\{B, B^\dagger\} + \{D, D^\dagger\} + \{B, D^\dagger\} + \{B^\dagger, D\} = 0 \tag{4.22b}$$

$$\{B, B^\dagger\} + \{D, D^\dagger\} - \{B, D^\dagger\} - \{B^\dagger, D\} = 0 \tag{4.22c}$$

$$\{B, B^\dagger\} - \{D, D^\dagger\} - \{B, D^\dagger\} + \{D, B^\dagger\} = -1 \tag{4.22d}$$

$$\{B, B^\dagger\} - \{D, D^\dagger\} + \{B, D^\dagger\} - \{D, B^\dagger\} = -1 \tag{4.22e}$$

with the solution

$$B^2 = D^2 = B^{\dagger 2} = D^{\dagger 2} = 0 \tag{4.23a}$$

$$\{B, D\} = \{B^\dagger, D\} = \{B, D^\dagger\} = \{B^\dagger, D^\dagger\} = 0 \tag{4.23b}$$

$$\{B^\dagger, B\} = -\frac{1}{2}, \qquad \{D^\dagger, D\} = \frac{1}{2} \tag{4.23c}$$

We now let $|0\rangle$ denote the fermionic vacuum for which

$$B|0\rangle = D|0\rangle = 0 \tag{4.24}$$

Defining $|0\rangle$ to have norm one, (4.23c) implies

$$\langle 0|BB^\dagger|0\rangle = -\frac{1}{2}, \qquad \langle 0|DD^\dagger|0\rangle = +\frac{1}{2} \tag{4.25}$$

so that

$$B^\dagger|0\rangle \neq 0, \qquad D^\dagger|0\rangle \neq 0 \tag{4.26}$$

The theory is thus seen to possess negative norm states in the fermionic sector. The existence of these negative norm states as free states of the fermionic part of $\mathcal{H}_{\text{BRST}}$ is, however, irrelevant to the existence of physical states in the orthogonal subspace of the Hilbert space.

In terms of annihilation and creation operators

$$\begin{aligned}\mathcal{H}_{\text{BRST}} = {} & \frac{1}{2}e^2(\Pi^-)^2 + \Pi^-(\partial_- A^-) - eA^-(\partial_-\phi) - e^2 A^+ A^- \\ & + \Pi^+\left(\frac{1}{e}\phi\right) + \frac{1}{2}(\Pi^+)^2 + 2(B^\dagger B + D^\dagger D)\end{aligned} \tag{4.27}$$

and the BRST charge operator Q is

$$\begin{aligned}Q = \int dx^- \, (i)[& B[(\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+) - i(\pi + \Pi^+ - \partial_-\phi - eA^+)] \\ & + D[(\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+) + i(\pi + \Pi^+ - \partial_-\phi - eA^+)]]\end{aligned} \tag{4.28}$$

Now because $Q|\psi\rangle = 0$, the set of states annihilated by Q contains not only the set of states for which (4.18) hold but also additional states for which

$$B|\psi\rangle = D|\psi\rangle = 0 \tag{4.29a}$$

$$\Pi^+|\psi\rangle \neq 0 \tag{4.29b}$$

$$[\pi - \partial_-\phi - eA^+]|\psi\rangle \neq 0 \tag{4.29c}$$

$$[\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+]|\psi\rangle \neq 0 \tag{4.29d}$$

The Hamiltonian is also invariant under the anti-BRST transformation given by

$$\hat{\bar{\delta}}\phi = -e\bar{c}, \qquad \hat{\bar{\delta}}A^+ = \partial_-\bar{c}, \qquad \hat{\bar{\delta}}A^- = \partial_+\bar{c}, \qquad \hat{\bar{\delta}}u = \partial_+\partial_+\bar{c} \tag{4.30a}$$

$$\hat{\bar{\delta}}\pi = \hat{\bar{\delta}}\Pi^+ = \hat{\bar{\delta}}\Pi^- = \hat{\bar{\delta}}\Pi_u = \hat{\bar{\delta}}\Pi_v = 0, \qquad \hat{\bar{\delta}}v = -e\partial_+\bar{c} \tag{4.30b}$$

$$\hat{\bar{\delta}}\bar{c} = 0, \qquad \hat{\bar{\delta}}c = -b, \qquad \hat{\bar{\delta}}b = 0 \tag{4.30c}$$

with the generator or anti-BRST charge

$$\begin{aligned}\bar{Q} & = \int dx^- \, [-i\bar{c}[\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+] + i(\partial_+\bar{c})[\pi + \Pi^+ - \partial_-\phi - eA^+]] \\ & = \int dx^- \, (-i)[B^\dagger[(\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+) - i(\pi + \Pi^+ - \partial_-\phi - eA^+)] \\ & \quad + D^\dagger[(\partial_-\Pi^- + e\,\partial_-\phi + e^2 A^+) + i(\pi + \Pi^+ - \partial_-\phi - eA^+)]]\end{aligned} \tag{4.31}$$

We also have

$$\partial_+ Q = [Q, H_{\mathrm{BRST}}] = 0 \tag{4.32a}$$

$$\partial_+ \bar{Q} = [\bar{Q}, H_{\mathrm{BRST}}] = 0 \tag{4.32b}$$

with

$$H_{\mathrm{BRST}} = \int dx^- \, \mathcal{H}_{\mathrm{BRST}} \tag{4.32c}$$

and we further impose the dual condition that both Q and $\bar{Q}$ annihilate physical states, implying that

$$Q|\psi\rangle = 0 \quad \text{and} \quad \bar{Q}|\psi\rangle = 0 \tag{4.33}$$

The states for which (4.18) holds satisfy both of these conditions, and in fact are the only states satisfying both of these conditions since, although with (4.23)

$$2(B^\dagger B + D^\dagger D) = -2(BB^\dagger + DD^\dagger) \tag{4.34}$$

there are no states of this operator with $B^\dagger|0\rangle = 0$ and $D^\dagger|0\rangle = 0$ [cf. (4.26)], and hence no free eigenstates of the fermionic part of H_{BRST} which are annihilated by each of B, $B^\dagger$, D, $D^\dagger$. Thus the only states satisfying (4.33) are those satisfying the constraints (3.11).

Further, the states for which (4.18) holds satisfy both the conditions (4.33), and in fact are the only states satisfying both of these conditions because in view of (4.21) one cannot have simultaneously c, $\partial_+ c$ and $\bar{c}$, $\partial_+ \bar{c}$ applied to $|\psi\rangle$ to give zero. Thus the only states satisfying (4.34) are those that satisfy the constraints of the theory (3.11) and they belong to the set of BRST-invariant and anti-BRST-invariant states.

Alternatively, one can understand the above point in terms of fermionic annihilation and creation operators as follows: The condition $Q|\psi\rangle = 0$ implies that the set of states annihilated by Q contains not only the states for which (4.18) holds but also additional states for which (4.29) holds. However, $\bar{Q}|\psi\rangle = 0$ guarantees that the set of states annihilated by $\bar{Q}$ contains only the states for which (4.18) holds, simply because $B^\dagger|\psi\rangle \neq 0$ and $D^\dagger|\psi\rangle \neq 0$. Thus in this alternative way also we see that the states satisfying $Q|\psi\rangle = \bar{Q}|\psi\rangle = 0$ (i.e., satisfying (4.33)) are only those states that satisfy the constraints of the theory and also that these states belong to the set of BRST-invariant and anti-BRST-invariant states.

5. SUMMARY AND DISCUSSION

In this work, we have considered the light-front Hamiltonian and BRST quantization of an abelian Higgs model in one space, one time dimension in the broken symmetry phase of the complex matter field Φ, where the phase ϕ of the complex matter field Φ carries the charge degree of freedom of the complex matter field and is in fact akin to the Goldstone boson. The theory in the broken symmetry phase can be thought of as a Higgs Lagrangian where the Higgs potential has been set to zero by freezing the complex matter field Φ at the degenerate minima of the potential. An important thing that happens here is that the vector gauge boson A^μ becomes massive in the broken symmetry phase and this mass generation takes place, perhaps, through a mechanism similar to the Higgs mechanism. Also the theory describes the low-lying excitations in the broken symmetry phase and therefore these studies could be relevant for the effective theories in the condensed matter.

In Kulshreshtha (2000b), the present theory has been studied in the usual instant-form of dynamics (conventional equal-time theory) on the hyperplanes $x^0 =$ constant. In the present work, the theory has been quantized on the light-front. The light-front quantization which studies the relativistic quantum dynamics of the physical system on the hyperplanes $\sqrt{2}x^+ \equiv \sqrt{2}t = (x^0 + x^1) =$ constant, also called the front-form theory, has several advantages over the conventional instant-form (equal-time) theory. In particular, for a light-front theory 7 out of 10 poincare generators are kinematical while the instant-form theory has only 6 kinematical generators (Dirac, 1949; Srivastava, 1998, 1999).

In our treatment, we have made the convention to regard the light-cone variable $x^+ \equiv t$ as the light-front time coordinate and the light-cone variable x^- has been treated as the longitudinal spatial coordinate. The temporal evolution of the system in x^+ is generated by the total Hamiltonian of the system (3.7). If we consider the invariant distance between two space-time points in (1+1)-dimension (Srivastava, 1998, 1999)

$$(x - y)^2 := (x^0 - y^0)^2 - (x^1 - y^1)^2 \qquad \text{(Instant-form)} \tag{5.1a}$$

$$(x - y)^2 := 2(x^+ - y^+)(x^- - y^-) \qquad \text{(Front-form)} \tag{5.1b}$$

then we find that in the instant-form, the points (on the hyperplanes $x^0 = y^0 =$ constant) have space-like separation except when they are coincident when it becomes light-like one. On the light-front, however, with $x^+ = y^+ =$ constant, the distance becomes independent of $(x^- - y^-)$ and the separation again becomes space-like. The light-front field theory therefore does not necessarily need to be local in x^-, even if the corresponding instant-form theory is formulated as a local one (Srivastava, 1998, 1999).

The nonvanishing equal-time commutators of the instant-form theory (cf. Kulshreshtha, 2000b) are nonlocal and nonvanishing for space-like distances and violate the microcausality principle (Srivastava, 1998, 1999). The nonvanishing equal-time, light-cone commutators of theory obtained in the present work (given by Eq. (3.20)), on the other hand, are nonlocal in the light-cone space variable x^- and nonvanishing only on the light-cone. There is therefore no conflict with the microcausality principle for the light-front theory unlike the case of the equal-time commutators in the instant-form theory. The constrained dynamics of the theory in the instant-form (Kulshreshtha, 2000b) reveals that the theory possesses a set of two first-class constraints, where one constraint is a primary constraint and the other one is a secondary Gauss-law constraint. The matrix of the Poission brackets of these two constraints is a singular matrix and therefore they form a set of first-class constraints, implying, in turn, that the corresponding theory is gauge invariant. The theory is indeed seen to possess a local vector gauge invariance given by (2.7) (Kulshreshtha, 2000b). The constrained dynamics of the theory in the light-front frame, as studied in the present work, reveals that the theory in the light-front frame possesses a set of three first-class constraints, where two constraints are primary and one is a secondary Gauss-law constraint. The matrix of the Poisson brackets of these three constraints is seen to be singular and therefore they together constitute a set of first-class constraints, implying that the theory is gauge invariant. The present theory is indeed also seen to posses a local vector gauge invariance given by (3.12) and correspondingly there exists a conserved local vector gauge current given by (3.13).

Now because the set of constraints of the theory is first-class, the Dirac quantization of the theory is possible only under some suitable gauge-fixing conditions or gauge choices. The choice of the suitable gauges for this purpose has to be such that these gauge-fixing conditions along with the original set of constraints of the theory form a set of second-class constraints so that the matrix of the Poisson brackets of all these constraints including the gauge-fixing conditions becomes nonsingular or invertible. Any such gauge-fixing conditions which make this possible are in principle acceptable gauge choices. However, one can choose the gauge-fixing conditions of ones choice based on some physical grounds or so to say choices of more physical importance. The gauge-fixing conditions that we have choosen in our present work for the Hamiltonian quantization of our theory, e.g., are $A^+ = 0$ or $\partial_- A^+ = 0$, both of which represent the temporal or time-axial gauge; and the other condition that we have used in our present work is $A^- = 0$ or $\partial_- A^- = 0$, both of which correspond to the so-called Coulomb gauge. These conditions are not only acceptable and consistent with the Dirac quantization procedure but are also physically interesting gauge choices representing the temporal/time-axial and the Coulomb gauges.

However, in the above Hamiltonian quantization of the theory under some gauge-fixing conditions one necessarily destroys the gauge invariance of the theory by fixing the gauge which converts the set of first-class constraints of the theory into a set of second-class one, by changing the matrix of the Poisson brackets of the constraints of the theory from a singular one into a nonsingular (invertible) one. In view of this, in order to achieve the quantization of our gauge-invariant theory, such that the gauge invariance of the theory is maintained even under gauge fixing, we go to a more generalized procedure called the BRST quantization (Becchi *et al.*, 1974; Henneaux, 1985; Kulshreshtha, 1998, 2000a,b, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin). The BRST quantization of the present gauge-invariant, light-front theory under some specific gauge choice (where a particular gauge has been choosen by us, only as an example for illustration and is not unique by any means) has finally been studied in Section 4. In this procedure, the BRST-quantized theory continues to possess the generalized or extended gauge invariance called the BRST symmetry even under the BRST gauge fixing (cf. Section 4) (Becchi *et al.*, 1974; Henneaux, 1985; Kulshreshtha, 1998, 2000a,b, in press-b; Kulshreshtha and Kulshreshtha, 1999, in press; Kulshreshtha *et al.*, 1993a–c, 1994a–d, 1995, 1999; Nemeschansky *et al.*, 1988; Tyutin).

ACKNOWLEDGMENTS

One of the authors (U.K.) thanks the CSIR, New Delhi, for the award of a Senior Research Associateship, which enabled her to carry out this research work.

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